

A 4 foils hydrofoiler regulation solution

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Abstract

The abstract should be one paragraph. It should summarise the problem, approach, and significance of the results.

1 Introduction

The idea of using a lifting wing with a dissymmetrical profile has, of course, triggered the early development of aviation, but has also been rapidly tested in a denser fluid, the water. The first occurrence of this usage is from 1861, when Thomas Moy experimented a towed structure with 3 lifting underwater surfaces, and witnessed the lift and drag reduction effects and their relation with flow velocity [REF]. The regulation of the lifting effect, which is still the key question, was originally solved with the use of self-regulated static structures as "ladder", "V", "L", "J" or "Y" shapes, where the lifting effect is reduced as the immersed part of the foiling structure decreases. These static structures are not optimal since they produce inner-forces and require structural elements that induce supplementary drag. Hence, the "T" foil is the optimal structure, but requires active regulation. This can be solved using a "frontal sensor wand" that regulates the foil's attack angle with a passive elastic transmission, as Moth hydrofoiler uses [REF]. Active regulation, i.e. electronically controlled, became a necessity as size and performance objective increased. The first approach was to control the incidence of a single stern foil to regulate altitude and pitch, with 2 passive foiling structures were installed at bow [REF] and while the boat velocity reaches a prescribed value. A full regulation of the atteignable degrees of freedom (altitude above water, roll and pitch), coping with different velocities has been done with 3 active foiling structures mounted on a catamaran ?? [REF Airbus].

This paper proposes a new control design for the attitude and altitude regulation of a 4-foils electric hydrofoiler, called Black-Pearl. The first section describe the system modelling and presents the notation that will

be used.

Previous realisations (state of the art)

Novelty of our approach (4 regulated foils)

2 System description

In the sequel, 'bold' notation is reserved for vectors and matrices, while scalars are written classically. Let $\{U\}$ be the NED (North-East-Down) universal coordinate frame and $\{B\}$ be the NED body frame, attached to the center of gravity of the system (when control angles, defined hereafter, are null). We also define a set of body-frames attached to each elements of the system, called element-frames and denoted $\{B_i\}$. In the sequel, the indices S and P stand for 'Starboard' and 'Port-board' and R and F mean 'Rear' and 'Front', respectively.

The ASV (Autonomous Surface Craft), called 'Black Pearl', is composed with 14 different elements (cf. Figure 1) :

- Two hulls, denoted $\mathcal{H}_S$ and $\mathcal{H}_P$, whose gravity centres are located in $\{B\}$ as $\mathbf{X}_{\mathcal{H}_S}^G$ and $\mathbf{X}_{\mathcal{H}_P}^G$.
- Four vertical legs, denoted $\mathcal{L}_{F,S}$, $\mathcal{L}_{R,S}$, $\mathcal{L}_{F,P}$, and $\mathcal{L}_{R,P}$, with their respective foils, denoted $\mathcal{F}_{F,S}$, $\mathcal{F}_{R,S}$, $\mathcal{F}_{F,P}$, and $\mathcal{F}_{R,P}$, rigidly attached at their respective leg's bottom. These legs are articulated with respect to the hulls, along the v axis of $\{B\}$ with angles $\delta_{F,S}$, $\delta_{R,S}$, $\delta_{F,P}$, and $\delta_{R,P}$. These articulations are located in $\{B\}$ at $\mathbf{X}_{F,S}^A$, $\mathbf{X}_{R,S}^A$, $\mathbf{X}_{F,P}^A$ and $\mathbf{X}_{R,P}^A$. The position of the center of mass of the 4 legs, in $\{B\}$, are expressed as $\mathbf{X}_{\mathcal{L}_{F,S}}^G(\delta_{F,S})$, $\mathbf{X}_{\mathcal{L}_{R,S}}^G(\delta_{R,S})$, $\mathbf{X}_{\mathcal{L}_{F,P}}^G(\delta_{F,P})$ and $\mathbf{X}_{\mathcal{L}_{R,P}}^G(\delta_{R,P})$. The center of mass of the 4 foils are denoted $\mathbf{X}_{\mathcal{F}_{F,S}}^G(\delta_{F,S})$, $\mathbf{X}_{\mathcal{F}_{R,S}}^G(\delta_{R,S})$, $\mathbf{X}_{\mathcal{F}_{F,P}}^G(\delta_{F,P})$ and $\mathbf{X}_{\mathcal{F}_{R,P}}^G(\delta_{R,P})$.
- A rudder, denoted $\mathcal{R}$, articulated along the w axis of $\{B\}$ with an angle δ_R . The position of the rudder's articulation is expressed as $\mathbf{X}_{\mathcal{R}}^A$, in $\{B\}$. The rudder's center of mass is located as $\mathbf{X}_{\mathcal{R}}^G(\delta_R)$, in $\{B\}$.
- A Thruster $\mathcal{T}$, rigidly attached at the bottom of a static vertical leg $\mathcal{L}_{\mathcal{T}}$, producing a thrust T . The center of mass of the leg is located at $\mathbf{X}_{\mathcal{L}_{\mathcal{T}}}^G$, in $\{B\}$ and the center of masse of the thruster is located at $\mathbf{X}_{\mathcal{T}}^G$.
- A load, denoted $\mathcal{L}$, representing the equivalent inertial effect of other system components (electronics, batteries, wires...) is mounted on the system at point $\mathbf{X}_{\mathcal{L}}^G$.

We also define a set of 14 element-frames $\{B_i\}$, where

$$i = \{\mathcal{H}_S, \mathcal{H}_P, \mathcal{F}_{F,S}, \mathcal{F}_{R,S}, \mathcal{F}_{F,P}, \mathcal{F}_{R,P}, \mathcal{L}_{F,S}, \mathcal{L}_{R,S}, \mathcal{L}_{F,P}, \mathcal{L}_{R,P}, \mathcal{R}, \mathcal{L}, \mathcal{L}_{\mathcal{T}}, \mathcal{T}\} \quad (1)$$

attached to the center of mass of each system's elements and described at Figure 1. The control variables are defined with the vector ζ as:

$$\zeta = [\delta_{F,S}, \delta_{R,S}, \delta_{F,P}, \delta_{R,P}, \delta_R, T]^T \quad (2)$$

2.1 Notation

In the sequel, $\boldsymbol{\eta}$ expresses the system state in $\{0\}$, $\boldsymbol{\nu}$ is the system velocities in $\{B\}$, as stated on Equations (3) and illustrated at Figure 1.

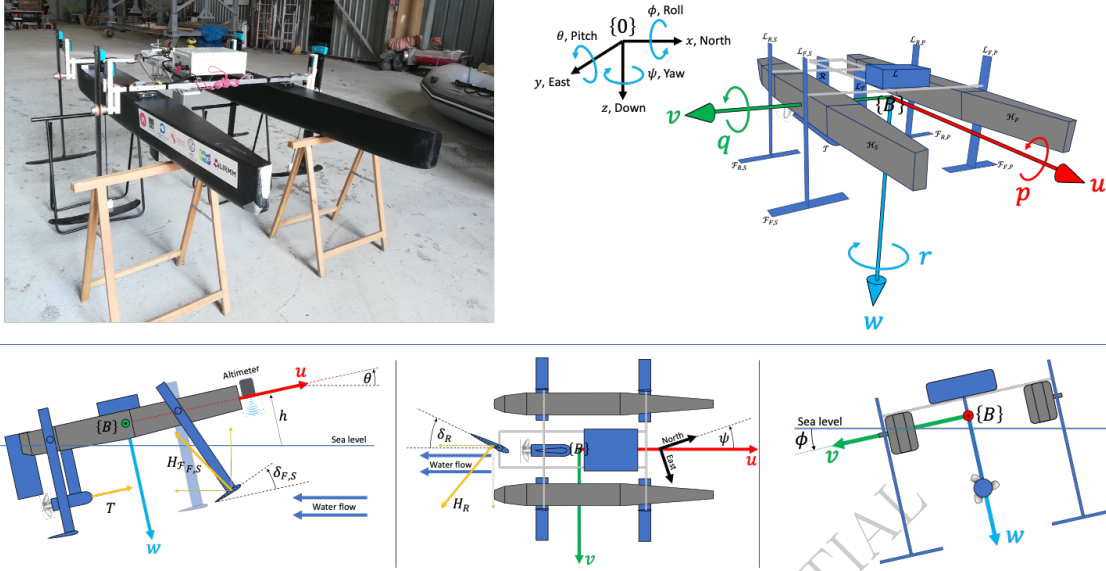


Figure 1: Notation and frames definition

$$\begin{aligned}
 \boldsymbol{\eta} &= [x, y, z, \phi, \theta, \psi]^T \\
 \mathbf{v} &= [u, v, w]^T \\
 \boldsymbol{\omega} &= [p, q, r]^T \\
 \boldsymbol{\nu} &= [\mathbf{v}^T, \boldsymbol{\omega}^T]^T
 \end{aligned} \tag{3}$$

Let $\mathbf{R}_A^B(\phi, \theta, \psi)$ denote the 3D rotation matrix from frame $\{A\}$ to frame $\{B\}$ with a relative attitude denoted by Euler angles ϕ (roll), θ (pitch) and ψ (yaw). Note that $\mathbf{R}_A^B$ is generally invertible and its inverse is denoted $\mathbf{R}_B^A = (\mathbf{R}_A^B)^{-1}$. The jacobian relation that links the Euler angle velocity $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ with body-frame rotational velocity w is denoted $\mathbf{J}_A^B(\phi, \theta)$.

$$\mathbf{R}_B^0(\phi, \theta, \psi) = \begin{bmatrix} \cos \psi \cos \theta & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi \\ \sin \psi \cos \theta & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \tag{4}$$

$$\mathbf{J}_B^0(\phi, \theta) = \begin{bmatrix} 1 & \sin(\phi) \tan(\theta) & \cos(\phi) \tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \frac{\sin(\phi)}{\cos(\theta)} & \frac{\cos(\phi)}{\cos(\theta)} \end{bmatrix} \tag{5}$$

The system kinematic model is then classically taken as :

$$\dot{\boldsymbol{\eta}}_u = \begin{bmatrix} \mathbf{R}_B^0(\phi, \theta, \psi) & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{J}_B^0(\phi, \theta) \end{bmatrix} \cdot \boldsymbol{\nu} \tag{6}$$

where $\mathbf{0}_{i \times j}$ denotes the null $(i \times j)$ matrix.

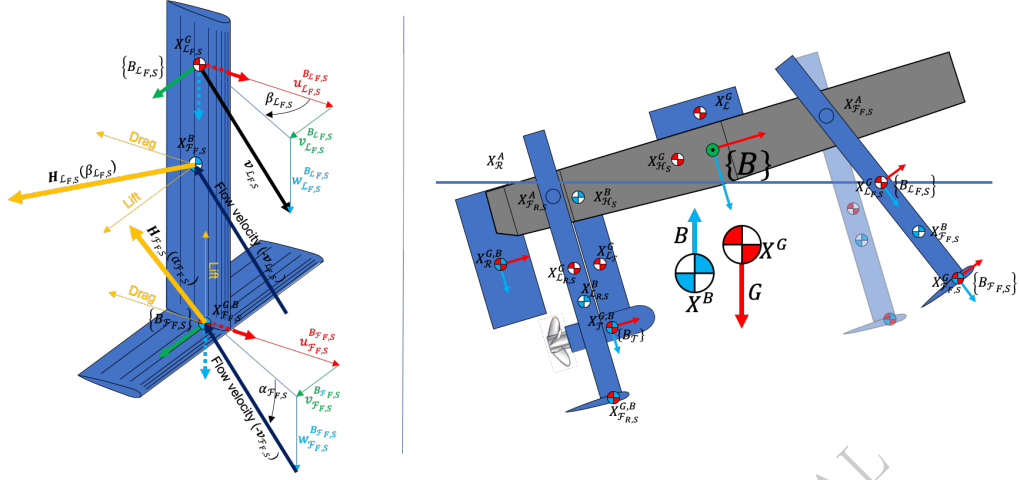


Figure 2: Forces applying on the system

2.2 Dynamic Modeling

A simplified dynamic model of the system is presented. Note that this model is used for control design and partial (simulation) validation. A more complete model is presented in [REF THESE LOIC].

2.2.1 Gravity and Buoyancy

The global system's gravity center location $\mathbf{X}^G$, in $\{B\}$, and the resulting weight force $\mathbf{G}$, can be computed as follows and is illustrated at Figure 2:

$$\mathbf{X}^G = \frac{\sum_i (m_i \cdot \mathbf{X}_i^G)}{M} \quad , \quad \mathbf{G} = \mathbf{R}_0^B \cdot [0, 0, M \cdot g]^T \quad (7)$$

where $g = 9.81 \text{ m/s}^2$ is the gravitational acceleration, m_i denotes the mass of the element i and $M = \sum_i m_i$ and i is defined at Equation (1).

Buoyancy acts on the immersed part of all the elements i of the system. It is expressed as the force $b_i = v_i \cdot \rho \cdot g$, where v_i denotes the volume of the immersed part of the element i and ρ is the water density. Its action point is located at the volumic center of the immersed part of the element i and is denoted $\mathbf{X}_i^B$, where i is defined at Equation (1). Hence the buoyancy can be computed as :

$$\mathbf{X}^B = \frac{\sum_i (b_i \cdot \mathbf{X}_i^B)}{B} \quad , \quad \mathbf{B} = \mathbf{R}_0^B \cdot [0, 0, -B \cdot \rho \cdot g]^T \quad (8)$$

where $B = \sum_i b_i$.

2.2.2 Hydrodynamic damping and thrust

Assuming that each system element i undergoes a laminar interaction with the fluid, hydrodynamic damping effect can be decomposed into two parts : Drag and Lift. Drag acts in the opposite direction of the flow

while Lift lives perpendicularly to it. For each element i , the magnitude of both these forces, denoted D_i and L_i depends on:

- The relative velocity of the water with respect to the immersed part of the element i . Considering that hydrodynamic forces apply at the center of buoyancy of i , we state that the relevant hydrodynamic velocity of i is denoted $\mathbf{v}_i = [u_i, v_i, w_i]^T$, in $\{B\}$, with $\mathbf{v}_i = \mathbf{v} + \boldsymbol{\omega} \times \mathbf{X}_i^B$. This element velocity, expressed in the local element-frame $\{B_i\}$ is denoted $\mathbf{v}_i^{B_i} = [u_i^{B_i}, v_i^{B_i}, w_i^{B_i}]^T$, allowing to define attack angle $\alpha_i = \arctan(w_i^{B_i}/u_i^{B_i})$ and side-slip angle $\beta_i = \arctan(v_i^{B_i}/u_i^{B_i})$, as illustrated in Figure 2.

We notice that :

- for the elements that are rigidly attached to the system, i.e. $j = \{\mathcal{H}_S, \mathcal{H}_P, \mathcal{L}, \mathcal{L}_T, \mathcal{T}\}$, $\mathbf{v}_j^{B_j} = \mathbf{R}_B^{B_j} \cdot \mathbf{v}_j$, with $\mathbf{R}_B^{B_j} = \mathbf{1}_3$ and where $\mathbf{1}_3$ denotes the (3×3) identity matrix.
- For the legs of the foils and the foils, i.e. $k = \{\mathcal{F}_{F,S}, \mathcal{F}_{R,S}, \mathcal{F}_{F,P}, \mathcal{F}_{R,P}, \mathcal{L}_{F,S}, \mathcal{L}_{R,S}, \mathcal{L}_{F,P}, \mathcal{L}_{R,P}\}$, $\mathbf{v}_k^{B_k} = \mathbf{R}_B^{B_k} \cdot \mathbf{v}_k$, where $\mathbf{R}_B^{B_k} = \mathbf{R}_B^{B_k}(0, \delta_k, 0)$.
- For the rudder, $\mathbf{v}_{\mathcal{R}}^{B_{\mathcal{R}}} = \mathbf{R}_B^{B_{\mathcal{R}}} \cdot \mathbf{v}_{\mathcal{R}}$, where $\mathbf{R}_B^{B_{\mathcal{R}}} = \mathbf{R}_B^{B_{\mathcal{R}}}(0, 0, \delta_R)$.
- The shape of i , its attack angle α_i and side-slip angle β_i with respect to the flow direction. This dependance is captured within the hydrodynamic diagonal coefficients matrix $\mathbf{C}_i(\alpha_i, \beta_i)$, decomposing the damping action along each components of $\mathbf{v}_i^{B_i}$, i.e.

$$\mathbf{C}_i = \begin{bmatrix} C_i^u & 0 & 0 \\ 0 & C_i^v & 0 \\ 0 & 0 & C_i^w \end{bmatrix} \quad (9)$$

where C_i^u is the drag coefficient of element i and C_i^v and C_i^w are either drag or lift coefficient, according to the function and situation of element i , as described at Figure 2 and Table 3.

- Hydrodynamic damping force, denoted $\mathbf{H}_i$ applies on each system component at their buoyancy center $\mathbf{X}_i^B$ as :

$$\mathbf{X}_i^B, \quad \mathbf{H}_i = -1/2 \cdot \rho \cdot S_i \cdot \mathbf{R}_{B_i}^B \cdot \mathbf{C}_i \cdot \mathbf{V}_i \cdot \mathbf{v}_i^{B_i} \quad (10)$$

where $\mathbf{V}_i$ is the selection matrix composed with the elements of $\mathbf{v}_i^{B_i}$ given at table XXX and S_i denotes the immersed surface of organ i .

- Thrust $\mathbf{T}$ induced by the thruster, and its input c_T , acts at point $\mathbf{X}_{\mathcal{T}}^G$. The thruster's characteristic $T(c_T)$ is given below.

$$\mathbf{X}_{\mathcal{T}}^G, \quad \mathbf{T} = [T, 0, 0]^T \quad (11)$$

Hence, total resulting force undergone by the system is expressed as vector $\mathbf{F}$:

$$\mathbf{F} = \mathbf{G} + \mathbf{B} + \mathbf{T} + \sum_i \mathbf{H}_i \quad (12)$$

These forces generate torques with respect to the body frame, resulting in vector $\boldsymbol{\Gamma}$, expressed as :

$$\boldsymbol{\Gamma} = \mathbf{B} \times (\mathbf{X}^B - \mathbf{X}^G) + \mathbf{T} \times (\mathbf{X}_{\mathcal{T}}^G - \mathbf{X}^G) + \sum_i (\mathbf{H}_i \times (\mathbf{X}_i^B - \mathbf{X}^G)) \quad (13)$$

for i defined at Equation (1). Finally, system dynamic model is written with respect to the body frame $\{B\}$ as

$$\begin{cases} \mathbf{F} &= [F_u, F_v, F_w]^T &= \mathbf{M} \cdot \dot{\mathbf{v}} \\ \mathbf{\Gamma} &= [\Gamma_p, \Gamma_q, \Gamma_r]^T &= \mathbf{I} \cdot \dot{\boldsymbol{\omega}} \end{cases} \quad (14)$$

where $\mathbf{M}$ and $\mathbf{I}$ are the mass and inertia matrices (including added mass and inertia coefficients).

2.3 Modelling the actuation system

The actuation system is relating forces and torques generated by the 6 actuators (4 foils, 1 rudder and 1 thruster) with the resulting action on the controlled degrees of freedom, expressed in the body frame. We choose to dispatch the control problem as follows :

- the 4 foils angulation regulate the system altitude (distance above water) roll and pitch angles.
- the rudder regulates the yaw angle,
- the thruster acts along u axis of $\{B\}$ and is set to a constant reasonable regime.

We extract from the dynamic model (Equations (12) and (13)) the terms related to the specific actions of the actuators along the regulated degrees of freedom.

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_0 + \mathbf{F}_{\mathcal{F}} + \mathbf{F}_{\mathcal{T}} \\ \mathbf{\Gamma} &= \mathbf{\Gamma}_0 + \mathbf{\Gamma}_{\mathcal{F}} + \mathbf{\Gamma}_{\mathcal{R}} \end{aligned} \quad (15)$$

where

$$\begin{aligned} \mathbf{F}_0 &= \mathbf{G} + \mathbf{B} + \sum_j \mathbf{H}_j + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \sum_f \mathbf{H}_f \\ \mathbf{\Gamma}_0 &= \mathbf{B} \times (\mathbf{X}^B - \mathbf{X}^G) + \mathbf{T} \times (\mathbf{X}_{\mathcal{T}}^G - \mathbf{X}^G) + \sum_k (\mathbf{H}_k \times (\mathbf{X}_k^B - \mathbf{X}_G)) + \\ &\quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \sum_f (\mathbf{H}_f \times (\mathbf{X}_f^B - \mathbf{X}_G)) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot (\mathbf{H}_R \times (\mathbf{X}_R^B - \mathbf{X}_G)) \end{aligned} \quad (16)$$

and

$$\begin{aligned} \mathbf{F}_{\mathcal{F}} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \sum_f \mathbf{H}_f = [0, 0, F_{\mathcal{F}}^w]^T \\ \mathbf{\Gamma}_{\mathcal{F}} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \sum_f (\mathbf{H}_f \times (\mathbf{X}_f^B - \mathbf{X}_G)) = [\Gamma_{\mathcal{F}}^p, \Gamma_{\mathcal{F}}^q, 0]^T \\ \mathbf{\Gamma}_{\mathcal{R}} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot (\mathbf{H}_R \times (\mathbf{X}_R^B - \mathbf{X}_G)) = [0, 0, \Gamma_{\mathcal{R}}^r]^T \\ \mathbf{F}_{\mathcal{T}} &= [F_{\mathcal{T}}^u, 0, 0]^T \end{aligned} \quad (17)$$

where $j = \{\mathcal{H}_S, \mathcal{H}_P, \mathcal{L}_{F,S}, \mathcal{L}_{R,S}, \mathcal{L}_{F,P}, \mathcal{L}_{R,P}, \mathcal{R}, \mathcal{L}, \mathcal{L}_T\}$, $k = \{\mathcal{H}_S, \mathcal{H}_P, \mathcal{L}_{F,S}, \mathcal{L}_{R,S}, \mathcal{L}_{F,P}, \mathcal{L}_{R,P}, \mathcal{L}, \mathcal{L}_T\}$ and $f = \{\mathcal{F}_{F,S}, \mathcal{F}_{R,S}, \mathcal{F}_{F,P}, \mathcal{F}_{R,P}\}$. $F_{\mathcal{F}}^w$ denotes the resulting force induced by the foils action along w axis, $\Gamma_{\mathcal{F}}^p$ and $\Gamma_{\mathcal{F}}^q$ denote the resulting torque induced by the foils around p and q axis, T denotes the thrust along u and $\Gamma_{\mathcal{R}}^r$ denotes the rudder's induced torque around w axis.

Hence, as a first stage, the actuation system is designed as the relation between the individual actuators and their resulting effect on the regulated degrees of freedom in $\{B\}$. The individual action of the foils along w in $\{B\}$ is denoted $F_{\mathcal{F}}^w$, written as :

$$F_{\mathcal{F}_f}^w = -1/2 \cdot \rho \cdot S_f \cdot \cos \delta_f \cdot (u_f^2 + w_f^2) \cdot C_f^w(\alpha_f) \quad (18)$$

where $\alpha_f = \arctan(u_f/w_f) + \delta_f$ and with $f = \{\mathcal{F}_{F,S}, \mathcal{F}_{R,S}, \mathcal{F}_{F,P}, \mathcal{F}_{R,P}\}$. $C_f^w(\alpha_f)$ is the lift coefficient of the foils given at table XXX. The resulting force and torque on the regulated d.o.f. of $\{B\}$ induced by the foils action is written as:

$$\begin{aligned} F_{\mathcal{F}}^w &= \sum_f F_{\mathcal{F}_f}^w \\ \Gamma_{\mathcal{F}}^p &= \sum_f \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \cdot (\mathbf{X}_f^B - \mathbf{X}_G) \cdot F_{\mathcal{F}_f}^w \\ \Gamma_{\mathcal{F}}^q &= \sum_f \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot (\mathbf{X}_f^B - \mathbf{X}_G) \cdot F_{\mathcal{F}_f}^w \end{aligned} \quad (19)$$

which can be written as :

$$\begin{bmatrix} F_{\mathcal{F}}^w \\ \Gamma_{\mathcal{F}}^p \\ \Gamma_{\mathcal{F}}^q \end{bmatrix} = \mathbf{A}_F \cdot \begin{bmatrix} C_{\mathcal{F}_{F,S}}^w(\alpha_{\mathcal{F}_{F,S}}) \\ C_{\mathcal{F}_{R,S}}^w(\alpha_{\mathcal{F}_{R,S}}) \\ C_{\mathcal{F}_{F,P}}^w(\alpha_{\mathcal{F}_{F,P}}) \\ C_{\mathcal{F}_{R,P}}^w(\alpha_{\mathcal{F}_{R,P}}) \end{bmatrix} \quad (20)$$

where $\mathbf{A}_F$ is a (3×4) matrix and denotes the foils' actuation system. One should note here the actuation redundancy expressed by the structure of $\mathbf{A}_F$. This property will be used in the sequel.

The contribution of the rudder action on r axis of $\{B\}$ is computed as follows:

$$\Gamma_{\mathcal{R}}^r = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot (\mathbf{X}_R^B - \mathbf{X}_G) \cdot \underbrace{(-1/2 \cdot \rho \cdot S_R(\eta) \cdot (u_R^2 + v_R^2) \cdot C_R^v(\alpha_R))}_{F_{\mathcal{F}_R}^v} = A_R \cdot C_R^v(\alpha_R) \quad (21)$$

where $\alpha_R = \arctan(u_R/v_R) + \delta_R$. Thruster's controlled action is along u of $\{B\}$ and simply expressed as :

$$F_{\mathcal{T}}^u = T \quad (22)$$

Finally, the global actuation system can be written as :

$$\begin{bmatrix} F_{\mathcal{T}}^u \\ F_{\mathcal{F}}^w \\ \Gamma_{\mathcal{F}}^p \\ \Gamma_{\mathcal{F}}^q \\ \Gamma_{\mathcal{R}}^r \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{0}_{1 \times 4} & 0 \\ \mathbf{0}_{3 \times 1} & \mathbf{A}_F & \mathbf{0}_{3 \times 1} \\ 0 & \mathbf{0}_{1 \times 4} & A_R \end{bmatrix} \cdot \begin{bmatrix} T \\ C_{\mathcal{F}_{F,S}}^w(\alpha_{\mathcal{F}_{F,S}}) \\ C_{\mathcal{F}_{R,S}}^w(\alpha_{\mathcal{F}_{R,S}}) \\ C_{\mathcal{F}_{F,P}}^w(\alpha_{\mathcal{F}_{F,P}}) \\ C_{\mathcal{F}_{R,P}}^w(\alpha_{\mathcal{F}_{R,P}}) \\ C_R^v(\alpha_R) \end{bmatrix} \quad (23)$$

The control design, addressed in the sequel, will require an inversion of the relation (23), which can be done on each decoupled components.

3 Control Design

As introduced earlier, the control objectives are decoupled with the following strategy :

- the 4 foils act along the 3 D.O.F w, p, q , w.r.t $\{B\}$, in order to regulate a desired altitude above water z_d , and 2 desired roll and pitch angles ϕ_d and θ_d , respectively.
- the rudder acts on the yaw dynamics and is open loop controlled,
- the thruster is set to a constant regime and produces the force T along u axis of $\{B\}$.

Consider the following expressions as desired closed-loop acceleration for the system :

$$\begin{aligned}\ddot{z}_c &= \ddot{z}_d + K_P^z \cdot (z_d - z) + K_I^z \cdot \int_{\tau=0}^t (z_d(\tau) - z(\tau))d\tau + K_D^z \cdot (\dot{z}_d - \dot{z}) \\ \ddot{\phi}_c &= \ddot{\phi}_d + K_P^\phi \cdot (\phi_d - \phi) + K_I^\phi \cdot \int_{\tau=0}^t (\phi_d(\tau) - \phi(\tau))d\tau + K_D^\phi \cdot (\dot{\phi}_d - \dot{\phi}) \\ \ddot{\theta}_c &= \ddot{\theta}_d + K_P^\theta \cdot (\theta_d - \theta) + K_I^\theta \cdot \int_{\tau=0}^t (\theta_d(\tau) - \theta(\tau))d\tau + K_D^\theta \cdot (\dot{\theta}_d - \dot{\theta})\end{aligned}\quad (24)$$

where $K_{P,I,D}^{z,\phi,\theta}$ are positive gains. Clearly, if the system tracks these desired acceleration, second order convergence of $z_d - z$, $\phi_d - \phi$ and $\theta_d - \theta$ is achieved. The expression of these references in the body frame is expressed as:

$$\begin{aligned}\dot{w}_c &= \frac{1}{\cos \theta \cdot \cos \phi} \cdot (\ddot{z}_c + \dot{u} \cdot \sin \theta - \dot{v} \cdot \cos \theta \cdot \sin \phi) \\ \dot{p}_c &= \ddot{\phi}_c - \dot{q} \cdot \sin \phi \cdot \tan \theta - \dot{r} \cdot \cos \phi \cdot \tan \theta \\ \dot{q}_c &= \frac{1}{\cos \phi} \cdot (\ddot{\theta}_c + \dot{r} \cdot \sin \phi)\end{aligned}\quad (25)$$

where it is assumed that $|\phi| < \pi/2$ and $|\theta| < \pi/2$ (Euler representation singularity). We now apply a classic 'computed torque' approach. The dynamic model along the w, p and q D.O.F. in $\{B\}$ can be extracted from (12) and (13). For the sake of simplicity of the control design, we assume that $\mathbf{M} = \text{diag}\{M_u, M_v, M_w\}$ and $\mathbf{I} = \text{diag}\{I_p, I_q, I_r\}$ are diagonal matrices. It is then direct to state that the desired acceleration references (25) are tracked by the system if :

$$\begin{bmatrix} C_{\mathcal{F}_{F,S}}^w(\alpha_{\mathcal{F}_{F,S}}) \\ C_{\mathcal{F}_{R,S}}^w(\alpha_{\mathcal{F}_{R,S}}) \\ C_{\mathcal{F}_{F,P}}^w(\alpha_{\mathcal{F}_{F,P}}) \\ C_{\mathcal{F}_{R,P}}^w(\alpha_{\mathcal{F}_{R,P}}) \end{bmatrix} = \mathbf{A}_{\mathcal{F}}^+ \cdot \left(\begin{bmatrix} M_w & 0 & 0 \\ 0 & I_p & 0 \\ 0 & 0 & I_q \end{bmatrix} \cdot \begin{bmatrix} \dot{w}_c \\ \dot{p}_c \\ \dot{q}_c \end{bmatrix} - \begin{bmatrix} \mathbf{F}_0^w \\ \mathbf{\Gamma}_0^p \\ \mathbf{\Gamma}_0^q \end{bmatrix} \right)\quad (26)$$

where $\mathbf{F}_0^w$ is the third component of $\mathbf{F}_0$, $\mathbf{\Gamma}_0^p$ is the first component of $\mathbf{\Gamma}_0$ and $\mathbf{\Gamma}_0^q$ is the second component of $\mathbf{\Gamma}_0$ ($\mathbf{F}_0$ and $\mathbf{\Gamma}_0$ being defined at Equation (15)) and $\mathbf{A}_{\mathcal{F}}^+$ is the Moore-Penrose pseudo-inverse of $\mathbf{A}_{\mathcal{F}}$. Assuming that $u > 0$, always, the desired foil angle is then computed as :

$$\delta_f = (C_f^w)^{-1} \left(\mathbf{A}_{\mathcal{F}}^+ \cdot \left(\begin{bmatrix} M_w & 0 & 0 \\ 0 & I_p & 0 \\ 0 & 0 & I_q \end{bmatrix} \cdot \begin{bmatrix} \dot{w}_c \\ \dot{p}_c \\ \dot{q}_c \end{bmatrix} - \begin{bmatrix} \mathbf{F}_0^w \\ \mathbf{\Gamma}_0^p \\ \mathbf{\Gamma}_0^q \end{bmatrix} \right) \right) - \arctan w_f / u_f\quad (27)$$

where $f = \{\mathcal{F}_{F,S}, \mathcal{F}_{R,S}, \mathcal{F}_{F,P}, \mathcal{F}_{R,P}\}$ and $(C_f^w)^{-1}$ denotes the inverse characteristic of the lift coefficient for the foil f . Since the four foils are identical, $C_{\mathcal{F}_{F,S}}^w = C_{\mathcal{F}_{R,S}}^w = C_{\mathcal{F}_{F,P}}^w = C_{\mathcal{F}_{R,P}}^w = C_{\mathcal{F}}^w$. $C_{\mathcal{F}}^w$ is given as a

piece-wise polynomial expression interpolating the look-up table of the chosen foil characteristics given by the Xfoil reference ¹. $(C_F^w)^{-1}$ is obtained similarly considering the inverted entries of the Xfoil's look-up table.

DIRECT EXPERIMENTATIONS ?

3.1 Guidance strategy design

The guidance system aims at shaping the error function in order to cope with system's capabilities and to shape a desired transitory regime. In our study, the strategy is threefold:

- Consider that foil efficiency increases with velocity. During take-off, when velocity u is small, a small error will easily push actuators to saturation. Assuming that the desired altitude above water is denoted h_d , the following expression implements this behavior.

$$\begin{aligned} z_d &= h_d \cdot \tanh(k_z \cdot u) \\ \tilde{z} &= z_d - z \end{aligned} \quad (28)$$

where $\tilde{z}$ stands for the altitude error and k_z is a positive gain shaping the stiffness of the tanh function.

- Regulate a transitory positive pitching during take-off. Here, we are inspired by the natural strategies of pitching positively the system to improve the take-off transitory regime, as kite-surfers do or motor trimming generally allows on rubber boats. As permanent regime is achieved, a null pitch angle is expected.

$$\begin{aligned} \theta_d &= \theta_{\max} \cdot \tanh(k_\theta \cdot \tilde{z}) \\ \tilde{\theta} &= \theta_d - \theta \end{aligned} \quad (29)$$

where $\tilde{\theta}$ denotes the pitching error, $\theta_{\max}$ is the maximum desired pitching angle and k_θ is a positive gain.

- Consider a 'banking turn' effect, as planes while changing heading, for passengers comfort. The desired banking angle is known (REF) and used as shown above.

$$\begin{aligned} \phi_d &= \frac{r \cdot v}{g} \\ \tilde{\phi} &= \phi_d - \phi \end{aligned} \quad (30)$$

where $\tilde{\phi}$ denotes the roll angle error.

Finally the foil control expression is written with:

$$\begin{aligned} \ddot{z}_c &= \ddot{z}_d + K_P^z \cdot \tilde{z} + K_I^z \cdot \int_0^t \tilde{z}(\tau) d\tau + K_D^z \cdot \dot{\tilde{z}} \\ \ddot{\phi}_c &= \ddot{\phi}_d + K_P^\phi \cdot \tilde{\phi} + K_I^\phi \cdot \int_0^t \tilde{\phi}(\tau) d\tau + K_D^\phi \cdot \dot{\tilde{\phi}} \\ \ddot{\theta}_c &= \ddot{\theta}_d + K_P^\theta \cdot \tilde{\theta} + K_I^\theta \cdot \int_0^t \tilde{\theta}(\tau) d\tau + K_D^\theta \cdot \dot{\tilde{\theta}} \end{aligned} \quad (31)$$

used in equations (25 - 27) to obtain the 4 desired foil angles δ_i .

¹<https://web.mit.edu/drela/Public/web/xfoil/>

3.2 Redundancy management

Since the actuation system is modeled with the matrix $\mathbf{C}_L$, which contains more lines than columns, the system has a potentially manageable redundancy. Indeed, 4 foils are used to regulate 3 D.O.F. Moreover, one can easily verify that

$$\text{rank}(\mathbf{C}_L) = 3 \quad (32)$$

since the choice of system parameters (cf. Table 3) and for any attainable values of foil angles δ_i . Hence, as done in [REF ACT], an element $\mathbf{N}_L$ of the kernel of $\mathbf{C}_L$ is identified and used as a projector in the null space of $\mathbf{C}_L$, as exposed in the sequel.

$$\mathbf{N}_L = \text{Ker}(\mathbf{C}_L) / \mathbf{C}_L \cdot \mathbf{N}_L \cdot r_m = 0 \quad (33)$$

where $\mathbf{N}_L$ is a (4×1) element of the null space of $\mathbf{C}_L$, and for any value of the scalar r_m . Hence, Equation (??) can be modify according to :

$$\mathbf{F}_L^c = \mathbf{C}_L^+ \cdot \begin{bmatrix} M_w \cdot \dot{w}_c - F_L^w \\ I_p \cdot \dot{p}_c - \Gamma_L^p \\ I_q \cdot \dot{q}_c - \Gamma_L^q \end{bmatrix} + \mathbf{N}_L \cdot r_m \quad (34)$$

and Equation (??) and (27) allow to compute the corresponding foil angles δ_i . By application of Equation (35), previous choice results in the following action, expressed in the $\{B\}$ frame as:

$$\begin{bmatrix} F_w^L \\ \Gamma_p^L \\ \Gamma_q^L \end{bmatrix} = \mathbf{C}_L \cdot \mathbf{C}_L^+ \cdot \begin{bmatrix} M_w \cdot \dot{w}_c - F_L^w \\ I_p \cdot \dot{p}_c - \Gamma_L^p \\ I_q \cdot \dot{q}_c - \Gamma_L^q \end{bmatrix} + \underbrace{\mathbf{C}_L \cdot \mathbf{N}_L \cdot r_m}_{=0} \quad (35)$$

which is identical to the previous desired control action. Hence, following the classic task function approach (REF) a specific value of r_m can be computed in order to respect an added criterion, without affecting the primary control objective (25). An example where a desired value is imposed on a foil is given below.

3.2.1 Imposing a desired value of a foil angle

Let's impose a desired constant value for the foil angle δ_i^* . The corresponding attack angle α_i^* is computed according to Equation (??) and (??) provides the induced lift force F_i^* . Let's now computed the corresponding r_m in order to produce desired lift force F_i^* on the i^{th} of $\mathbf{F}_L^c$, by inversion of relation (34) as :

$$r_m = (\mathbf{N}_L(i))^{-1} \cdot \left(F_i^* - \mathbf{C}_L(i, :) \cdot \begin{bmatrix} M_w \cdot \dot{w}_c - F_L^w \\ I_p \cdot \dot{p}_c - \Gamma_L^p \\ I_q \cdot \dot{q}_c - \Gamma_L^q \end{bmatrix} \right) \quad (36)$$

where $\mathbf{N}_L(i)$ is the i^{th} component of the (1×4) matrix $\mathbf{N}_L$ and $\mathbf{C}_L(i, :)$ is the i^{th} line of $\mathbf{C}_L$.

Table 1: Control gains used in simulation

	K_P	K_I	K_D
z_c	100	500	10
ϕ_c	800	0	100
θ_c	800	100	200

Table 2: Parameters of the guidance functions

k_z	$\theta_{\max}$	k_θ
0.8	0.2	5

4 Simulation results

In order to illustrate the performance of the solution proposed, we performed simulations with the parameters reported at table 3, where it is assumed that the charge does not have any hydrodynamic effect. The foils have the Eppler 817 profile reported at Figure XXX.

In the following, we present simulations results with an identical scenario : for the first 10 seconds, the thruster produces a constant force $T = 30N$, then for $t = 10s$ to $t = 20s$ the rudder is used with $\delta_R = 0.3rad$, and for $t = 20s$ to $t = 30s$ the rudder is oriented to $\delta_R = -0.3rad$.

In order to show the advantage of using a foiling structure, we first perform a simulation of the system without foiling structures. State evolution are given at Figure 3-up. The system reaches a constant velocity $u_\infty = 1.63m/s$. One should notice that the hull is a simple flat floating foam with very poor hydrodynamic characteristic. The torque produced by the thrust is clearly visible and induces a positive static pitch angle $\theta_\infty = 0.054rad$. At least the system performed the trajectory depicted at Figure 3-down.

The second simulation shows the system behavior without guidance function, with a constant $z_d = -0.2m$, $\theta_d = 0$ and $\phi_d = 0$. The control gains have been chosen according to table 1. The results are shown at Figure 4.

The analysis of Figure 4 shows that the system is achieving regulation during the first 10 seconds, while the rudder is fixed to $\delta_r = 0$. The velocity reaches a velocity of $u = 7.1 m/s$, showing the efficiency of the foiling strategy. The system takes-off at $t = 4.13 s$, after an actuation saturation period as the Foils activity Figures indicates, 4-down-right. The first rudder action ($\delta_R = 0.3 rad$ at $t = 10 s$) induces a roll angle dynamic which produces a constant roll of $\phi = 7e^{-3}rad$, while the system maintains regulation. The second rudder change ($\delta_R = -0.3 rad$ at $t = 20 s$) induces a bigger effects which pushes actuation to saturation and a bad system response to this disturbance. Finally, the system trajectory is shown at Figure 4-down-left.

The third simulation includes the guidance functions of equations (28), (29) and (30), with the control gains of Table 1 and guidance parameters of Table 2. Results are shown at Figure 5.

The system follows the guidance references, depicted in red at Figure 5-up. As shown on the actuation activity Figure 5-down-right, actuators are pushed to saturation only at the beginning, while the forward velocity u is small. Clearly the system response is better as the guidance references are followed. At least system trajectory is shown at Figure 5-down-left and exhibits a longer trajectory as the system efficiency is improved. Notice also that the achieved curvature is also improved, as one could expected from the banking strategy.

The fourth simulation implements the redundancy management proposed at section XXX. We follow previous

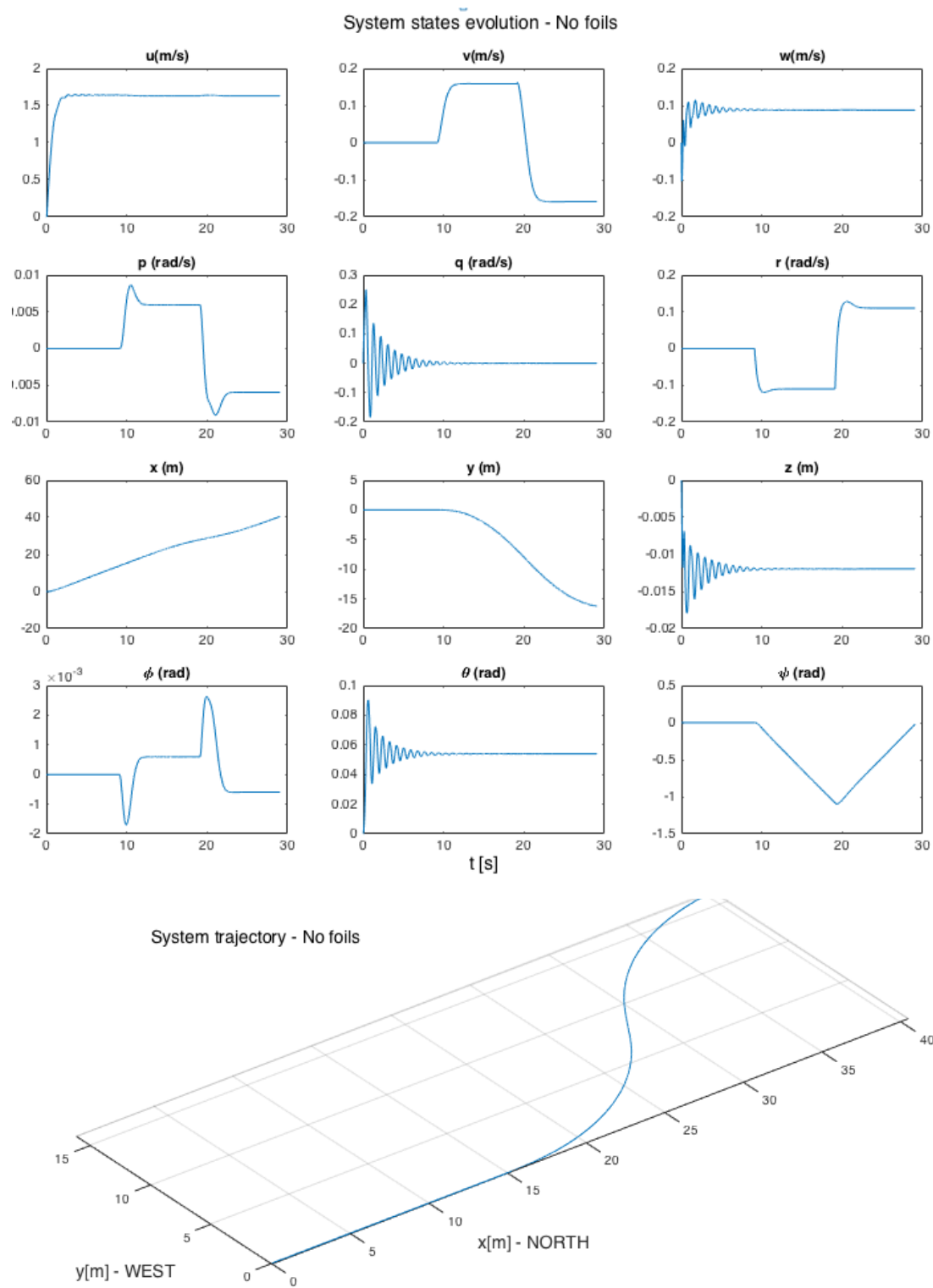


Figure 3: System states evolution (up) and trajectory (down), without foils

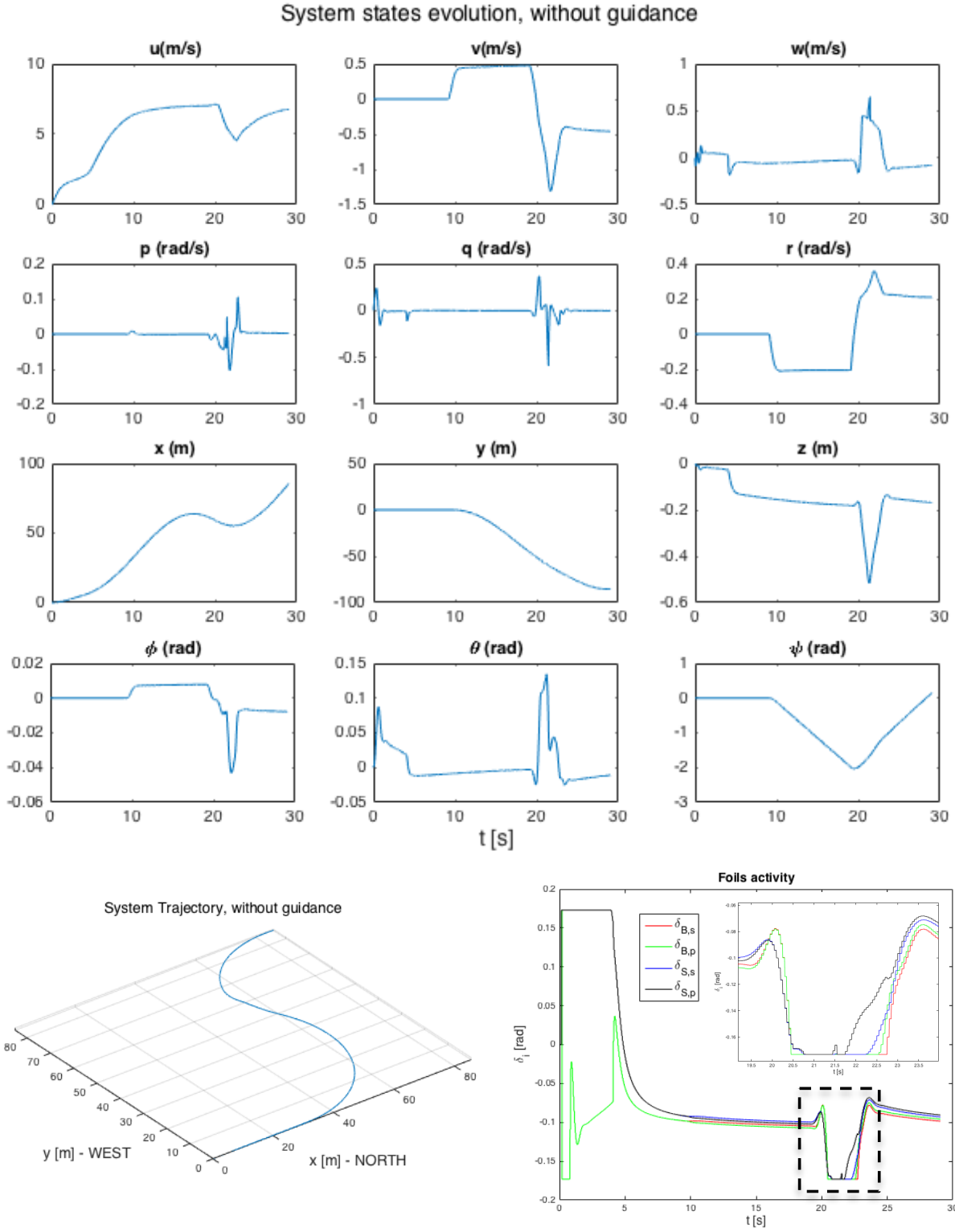


Figure 4: System states evolution (up), trajectory (down-left) and actuator activity (down-right), without guidance

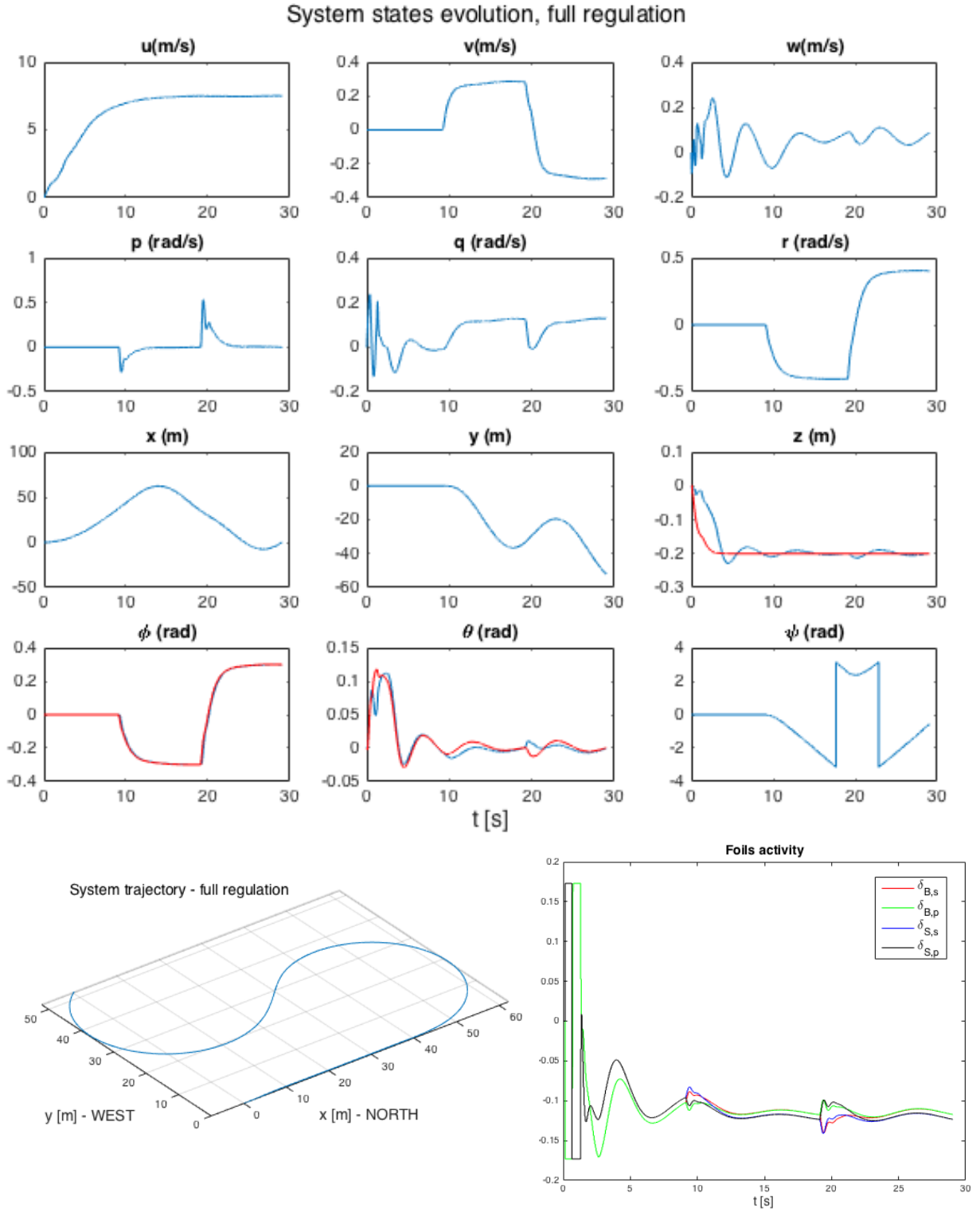


Figure 5: System states evolution (up), trajectory (down-left) and actuator activity (down-right), with guidance

Table 3: Systems parameters

	Pos / $\{B\}$ (m)	C_D (-)	Surf. Proj.(m ²)	mass (kg)	volume (m ³)
Hull	$[0, 0, 0]$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}$	$\begin{bmatrix} 0.033 & 0 & 0 \\ 0 & 0.066 & 0 \\ 0 & 0 & 0.605 \end{bmatrix}$	0.9	0.0363
Foil's legs	$x_L = 0.3$ $y_L = 0.275$ $z_L = 0.1$ $L_L = 0.85$	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.0085 & 0 & 0 \\ 0 & 0.085 & 0 \\ 0 & 0 & 0.0008 \end{bmatrix}$	0.7	$6.6e^{-4}$
Foils	$L_F = 0.525$	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.0036 & 0 & 0 \\ 0 & 0.0008 & 0 \\ 0 & 0 & 0.036 \end{bmatrix}$	0.1	$2.5e^{-4}$
Thruster's leg	$x_T = -0.2$ $z_T = 0.12$ $L_T = 0.24$	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.0024 & 0 & 0 \\ 0 & 0.048 & 0 \\ 0 & 0 & 0.021 \end{bmatrix}$	0.8	$6.6e^{-4}$
Thruster	-	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.0038 & 0 & 0 \\ 0 & 0.0252 & 0 \\ 0 & 0 & 0.0252 \end{bmatrix}$	3	0.0014
Rudder	$x_R = -0.6$ $z_R = 0.2$ $L_R = 0.4$	$\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$	$\begin{bmatrix} 0.0002 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.0001 \end{bmatrix}$	1	$3.1e^{-5}$
Charge	$[0, 0, 0]$	-	-	10	-

scenario with a null r_m up to $t = 6s$, where a desired $\delta_{B,p}^* = -0.1$ rad is imposed on bow-portside foil . Then Equation (36) is implemented and used as in (34). Results are shown at Figure 6

Analyzing the results of Figure 6, it is clear that, as expected, the constraint $\delta_{B,pB,p}^* 0 - 0.1$ rad does not affect the system response. Nevertheless, one should note that the constraint has been chosen close to the regulation regime of previous simulation. Indeed, this type of constraint can easily pushes the other actuators to saturation, hence degrading the regulation capabilities.

5 Experimentations

We performed experimental validation on the system... To be included when experimentation will be completed

6 Conclusion and future work

What we did : regulate a 4-foiler system, include guidance law for take off and curve-banking. What remains to be done : robustness of the controller with a moving mass onboard...

Acknowledgments

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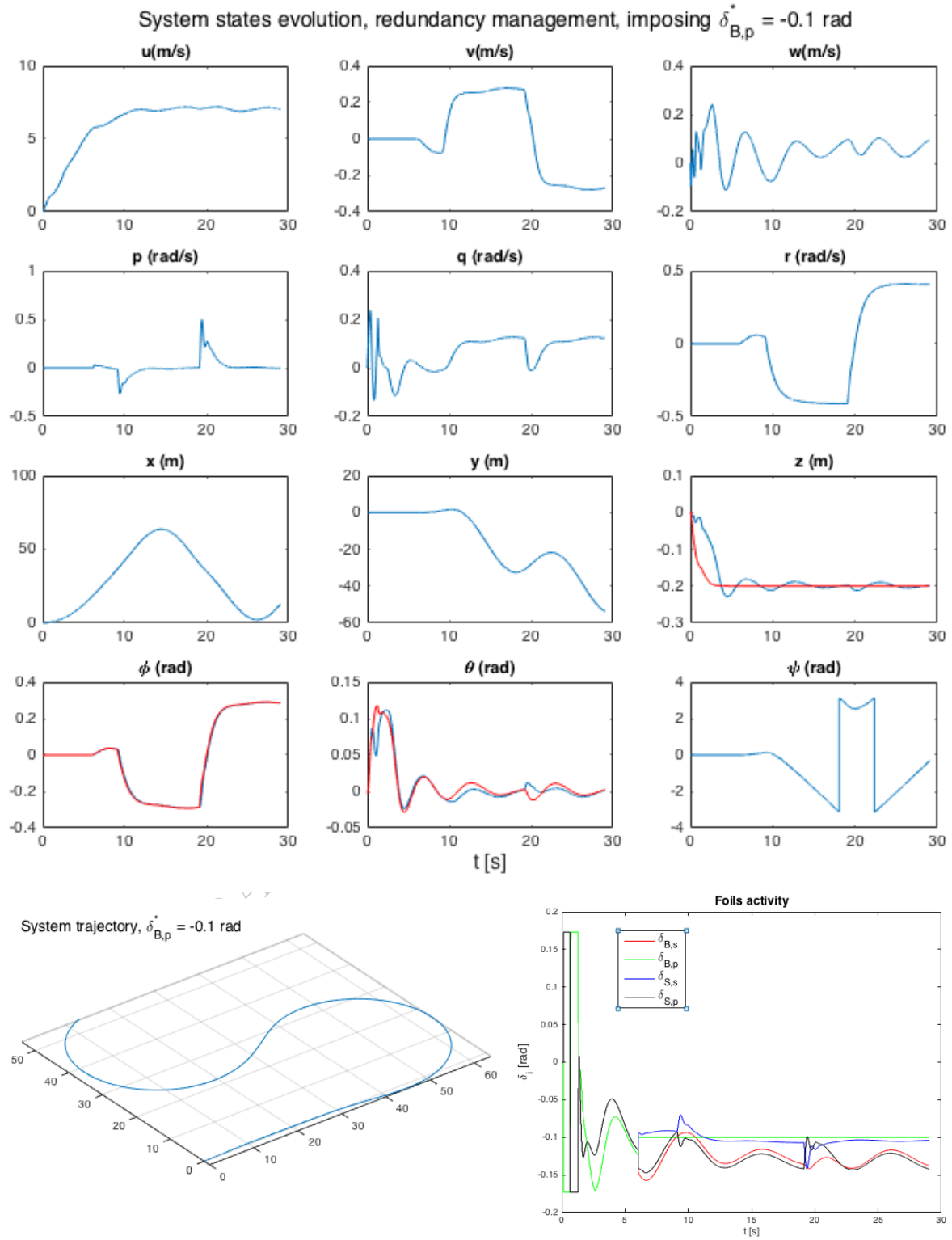


Figure 6: System states evolution (up), trajectory (down-left) and actuator activity (down-right), imposing $\delta_{B,p}^* = -0.1$ rad